Maple 2018. 2 Integration Test Results
on the problems in "1 Algebraic functions/1.1 Binomial products/1.1.4 Improper"
Test results for the 123 problems in "1.1.4.2 ( $c x)^{\wedge} m\left(a x^{\wedge} j+b x^{\wedge} n\right)^{\wedge} p . t x t^{\prime \prime}$
Problem 27: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{x^{4}}{\sqrt{b x^{4}+a x}} \mathrm{~d} x
$$

Optimal (type 3, 43 leaves, 3 steps):

$$
-\frac{a \operatorname{arctanh}\left(\frac{x^{2} \sqrt{b}}{\sqrt{b x^{4}+a x}}\right)}{3 b^{3 / 2}}+\frac{x \sqrt{b x^{4}+a x}}{3 b}
$$

Result (type 4, 996 leaves):
$\frac{x \sqrt{b x^{4}+a x}}{3 b}-\left(a\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}(x}\right.$

$$
\left.-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)
$$

$$
\sqrt[2]{ } \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}
$$

$$
\begin{aligned}
& \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}} 11 / b\left(-a b^{2}\right)^{1 /} \\
& { }^{3} \text { EllipticF } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}, \\
& \left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)}\right)-\frac{1}{b}\left(\left(-a b^{2}\right)^{1 /}\right. \\
& { }^{3} \text { EllipticPi } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}, \frac{-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}}{-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}},} \\
& \left.\sqrt{\left.\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)\right)}\right) \quad\left(\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)( \right. \\
& \left.\left.-a b^{2}\right)^{1 / 3} \sqrt{b x\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)
\end{aligned}
$$

Problem 28: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5}}{\sqrt{b x^{4}+a x}} \mathrm{~d} x
$$

Optimal(type 4, 521 leaves, 6 steps):

Result(type 4, 1078 leaves):

$$
\frac{x^{2} \sqrt{b x^{4}+a x}}{4 b}-\left(5 a \left(x\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)+\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right.\right.\right.
$$

$$
\left.-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}(x
$$

$$
\left.-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)
$$

$$
\begin{aligned}
& -\frac{5 a x\left(b x^{3}+a\right)(1+\sqrt{3})}{8 b^{5 / 3}\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right) \sqrt{b x^{4}+a x}}+\frac{x^{2} \sqrt{b x^{4}+a x}}{4 b}+\left(5 3 ^ { 1 / 4 } a ^ { 4 / 3 } x ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1+\sqrt{3})\right) \text { EllipticE }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right) /\left(8 \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.-\sqrt{3})) b^{5 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)+\left(5 a ^ { 4 / 3 } x ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1+\sqrt{3})\right) \text { EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)(1-\sqrt{3}) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) / \\
& \left(48\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right) b^{5 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt[2]{ } \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}} \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}} \\
& 1 /\left(\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}\right)\left(\left(\frac{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}}{b}\right.\right. \\
& \left.+\frac{\left(-a b^{2}\right)^{2 / 3}}{b^{2}}\right) b \text { EllipticF } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}, \\
& \left.\left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)}\right)+\frac{1}{\left(-a b^{2}\right)^{1 / 3}}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right. \\
& \left.+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \text { EllipticE } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}, \\
& \left.\left.\left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right.}\right) b\right)\right) \text { )} \\
& \left(8 b \sqrt{b x\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)
\end{aligned}
$$

Problem 29: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{2}}{\sqrt{b x^{4}+a x}} d x
$$

Optimal(type 4, 500 leaves, 5 steps):

Result(type 4, 1053 leaves):

$$
\left(x\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)+\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right.\right.
$$

$$
\left.-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}(x
$$

$$
\begin{aligned}
& \frac{x\left(b x^{3}+a\right)(1+\sqrt{3})}{b^{2 / 3}\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right) \sqrt{b x^{4}+a x}}-\left(3 ^ { 1 / 4 } a ^ { 1 / 3 } x ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.+\sqrt{3}) \text { ) EllipticE }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right) /\left(\left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.-\sqrt{3})) b^{2 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)-\left(a ^ { 1 / 3 } x ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}+b^{1 / 3} x\left(1^{1}\right.\right.\right. \\
& \left.+\sqrt{3}) \text { ) EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)(1-\sqrt{3}) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) /\left(6 \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1-\sqrt{3})\right) b^{2 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

$$
\left.-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)
$$

$$
\begin{aligned}
& 2 \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)} \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}} \begin{array}{l}
1 /\left(\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}\right) \\
1 /\left(\frac{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}}{b}\right.
\end{array}
\end{aligned}
$$

$$
\left.+\frac{\left(-a b^{2}\right)^{2 / 3}}{b^{2}}\right) b \text { EllipticF } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}
$$

$$
\left.\left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)}\right)+\frac{1}{\left(-a b^{2}\right)^{1 / 3}}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right.
$$

$$
\left.+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \text { EllipticE } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}
$$

$$
\begin{aligned}
& \sqrt{\left.\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right) b\right)} \sqrt{b x\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}
\end{aligned}
$$

Problem 30: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{x \sqrt{b x^{4}+a x}} \mathrm{~d} x
$$

Optimal(type 4, 523 leaves, 6 steps):

$$
\begin{aligned}
& \frac{2 b^{1 / 3} x\left(b x^{3}+a\right)(1+\sqrt{3})}{a\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right) \sqrt{b x^{4}+a x}}-\frac{2 \sqrt{b x^{4}+a x}}{a x}-\left(2 3 ^ { 1 / 4 } b ^ { 1 / 3 } x ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\sqrt{3})) a^{2 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)-\left(b^{1 / 3 x\left(a^{1 / 3}+b^{1 / 3} x\right) \sqrt{\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\left(a^{1 / 3}+b^{1 / 3} x(1\right.}\right. \\
& \left.+\sqrt{3}) \text { ) EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)(1-\sqrt{3}) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) /\left(3 \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1-\sqrt{3})\right) a^{2 / 3} \sqrt{b x^{4}+a x} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

Result(type 4, 1082 leaves):
$-\frac{2\left(b x^{3}+a\right)}{a \sqrt{x\left(b x^{3}+a\right)}}+\left(2 b\left(x\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)+\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right.\right.\right.$

$$
\begin{aligned}
& \left.-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}(x) \\
& \left.-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)
\end{aligned}
$$

$$
\begin{aligned}
& 2 \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)} \sqrt{\frac{\left(-a b^{2}\right)^{1 / 3}\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{b\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}} \begin{array}{l}
\sqrt{2}) \\
1 /\left(\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}\right)\left(\sqrt{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(-a b^{2}\right)^{1 / 3}}\right. \\
b
\end{array}
\end{aligned}
$$

$$
\left.+\frac{\left(-a b^{2}\right)^{2 / 3}}{b^{2}}\right) b \text { EllipticF } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}
$$

$$
\left.\left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)}\right)+\frac{1}{\left(-a b^{2}\right)^{1 / 3}}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}\right.
$$

Problem 31: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{1}{(b \sqrt{x}+a x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 2, 19 leaves, 1 step):

$$
\frac{4 \sqrt{x}}{b \sqrt{b \sqrt{x}+a x}}
$$

Result(type 3, 403 leaves):

$$
\frac{1}{\sqrt{\sqrt{x}(b+a \sqrt{x})} b^{2} \sqrt{a}(b+a \sqrt{x})^{2}}\left(\sqrt { b \sqrt { x } + a x } \left(2 a^{5 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} x+2 a^{5 / 2} \sqrt{b \sqrt{x}+a x} x-4 a^{3} / 2(\sqrt{x}(b+a \sqrt{x}))^{3 / 2}\right.\right.
$$

$$
+4 a^{3} / 2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{x} b+4 a^{3} / 2 \sqrt{b \sqrt{x}+a x} \sqrt{x} b+2 \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) \sqrt{x} a b^{2}
$$

$$
+\ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) x a^{2} b-2 \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) \sqrt{x} a b^{2}
$$

$$
-\ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) x a^{2} b+2 \sqrt{a} b^{2} \sqrt{\sqrt{x}(b+a \sqrt{x})}+2 \sqrt{a} \sqrt{b \sqrt{x}+a x} b^{2}
$$

$$
\left.\left.+\ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b^{3}-\ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b^{3}\right)\right)
$$

$$
\begin{aligned}
& \left.+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) \text { EllipticE } \sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)}}, \\
& \left.\left.\left.\sqrt{\left(\frac{\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}{\left(\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(\frac{3\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right.}\right) b\right)\right) \text { ) } \\
& \left(a \sqrt{b x\left(x-\frac{\left(-a b^{2}\right)^{1 / 3}}{b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}+\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)\left(x+\frac{\left(-a b^{2}\right)^{1 / 3}}{2 b}-\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}}{2 b}\right)}\right)
\end{aligned}
$$

Problem 34: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{x}}{\sqrt{b \sqrt{x}+a x}} \mathrm{~d} x
$$

Optimal (type 3, 63 leaves, 5 steps):

$$
\frac{3 b^{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b \sqrt{x}+a x}}\right)}{2 a^{5 / 2}}-\frac{3 b \sqrt{b \sqrt{x}+a x}}{2 a^{2}}+\frac{\sqrt{x} \sqrt{b \sqrt{x}+a x}}{a}
$$

Result(type 3, 163 leaves):

$$
\begin{aligned}
& \frac{1}{4 \sqrt{\sqrt{x}(b+a \sqrt{x})} a^{9 / 2}}\left(\sqrt { b \sqrt { x } + a x } \left(4 \sqrt{b \sqrt{x}+a x} \sqrt{x} a^{7 / 2}-8 b \sqrt{\sqrt{x}(b+a \sqrt{x})} a^{5 / 2}+2 \sqrt{b \sqrt{x}+a x} b a^{5 / 2}\right.\right. \\
& \left.\left.-b^{2} \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) a^{2}+4 b^{2} \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) a^{2}\right)\right)
\end{aligned}
$$

Problem 35: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{x} \sqrt{b \sqrt{x}+a x}} \mathrm{~d} x
$$

Optimal(type 3, 24 leaves, 3 steps):

$$
\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b \sqrt{x}+a x}}\right)}{\sqrt{a}}
$$

Result(type 3, 135 leaves):

$$
\begin{aligned}
& -\frac{1}{\sqrt{\sqrt{x}(b+a \sqrt{x})} b \sqrt{a}}\left(\sqrt { b \sqrt { x } + a x } \left(2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}-2 \sqrt{b \sqrt{x}+a x} \sqrt{a}-\ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b\right.\right. \\
& \left.\left.-\ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b\right)\right)
\end{aligned}
$$

Problem 36: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int_{x^{5} / 2 \sqrt{b \sqrt{x}+a x}} \mathrm{~d} x
$$

Optimal(type 2, 84 leaves, 4 steps):

$$
-\frac{4 \sqrt{b \sqrt{x}+a x}}{7 b x^{2}}+\frac{24 a \sqrt{b \sqrt{x}+a x}}{35 b^{2} x^{3 / 2}}-\frac{32 a^{2} \sqrt{b \sqrt{x}+a x}}{35 b^{3} x}+\frac{64 a^{3} \sqrt{b \sqrt{x}+a x}}{35 b^{4} \sqrt{x}}
$$

Result(type 3, 231 leaves):

$$
\begin{aligned}
& -\frac{1}{35 \sqrt{\sqrt{x}(b+a \sqrt{x})} b^{5} x^{9 / 2}}\left(\sqrt { b \sqrt { x } + a x } \left(35 a^{7 / 2} \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b x^{9 / 2}\right.\right. \\
& -35 a^{7 / 2} \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b x^{9 / 2}+70 a^{4} \sqrt{\sqrt{x}(b+a \sqrt{x})} x^{9 / 2}-140 a^{3}(b \sqrt{x}+a x)^{3 / 2} x^{7 / 2} \\
& \left.\left.+70 a^{4} \sqrt{b \sqrt{x}+a x} x^{9 / 2}-44(b \sqrt{x}+a x)^{3 / 2} x^{5 / 2} a b^{2}+76 a^{2}(b \sqrt{x}+a x)^{3 / 2} b x^{3}+20(b \sqrt{x}+a x)^{3 / 2} x^{2} b^{3}\right)\right)
\end{aligned}
$$

Problem 37: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5 / 2}}{(b \sqrt{x}+a x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 125 leaves, 8 steps):

$$
\frac{315 b^{4} \operatorname{arctanh}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b \sqrt{x}+a x}}\right)}{32 a^{11 / 2}}-\frac{4 x^{5 / 2}}{a \sqrt{b \sqrt{x}+a x}}-\frac{315 b^{3} \sqrt{b \sqrt{x}+a x}}{32 a^{5}}-\frac{21 b x \sqrt{b \sqrt{x}+a x}}{4 a^{3}}+\frac{9 x^{3 / 2} \sqrt{b \sqrt{x}+a x}}{2 a^{2}}+\frac{105 b^{2} \sqrt{x} \sqrt{b \sqrt{x}+a x}}{16 a^{4}}
$$

Result(type 3, 530 leaves):

$$
\begin{aligned}
& \frac{1}{64 a^{21 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})}(b+a \sqrt{x})^{2}}\left(\sqrt { b \sqrt { x } + a x } \left(32 a^{19 / 2}(b \sqrt{x}+a x)^{3 / 2} x^{3 / 2}-48 a^{17 / 2}(b \sqrt{x}+a x)^{3 / 2} x b\right.\right. \\
& +276 a^{17 / 2 \sqrt{b \sqrt{x}+a x} x^{3} / 2 b^{2}-768 a^{15 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} x b^{3}-192 a^{15 / 2}(b \sqrt{x}+a x)^{3 / 2} \sqrt{x} b^{2}+690 a^{15 / 2} \sqrt{b \sqrt{x}+a x} x b^{3}} \\
& +256 b^{3} a^{13 / 2(\sqrt{x}(b+a \sqrt{x}))^{3 / 2}-1536 a^{13 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{x} b^{4}-112 a^{13 / 2}(b \sqrt{x}+a x)^{3 / 2} b^{3}+552 a^{13 / 2 \sqrt{b \sqrt{x}+a x} \sqrt{x} b^{4}}} \begin{array}{l}
-768 a^{11 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} b^{5}+138 a^{11 / 2 \sqrt{b \sqrt{x}+a x} b^{5}-69 \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) x a^{7} b^{4}} \\
+384 \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) x a^{7} b^{4}-138 \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) \sqrt{x} a^{6} b^{5}
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& +768 \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) \sqrt{x} a^{6} b^{5}-69 \ln \left(\frac{2 \sqrt{b \sqrt{x}+a x} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) a^{5} b^{6} \\
& \left.\left.+384 \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) a^{5} b^{6}\right)\right)
\end{aligned}
$$

Problem 38: Result more than twice size of optimal antiderivative.

$$
\int \frac{\sqrt{x}}{(b \sqrt{x}+a x)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 44 leaves, 4 steps):

$$
\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{a} \sqrt{x}}{\sqrt{b \sqrt{x}+a x}}\right)}{a^{3 / 2}}-\frac{4 \sqrt{x}}{a \sqrt{b \sqrt{x}+a x}}
$$

Result(type 3, 239 leaves):

$$
\begin{aligned}
& -\frac{1}{a^{3 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} b(b+a \sqrt{x})^{2}}\left(2 \sqrt { b \sqrt { x } + a x } \left(2 a^{5 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} x-2 a^{3 / 2}(\sqrt{x}(b+a \sqrt{x}))^{3 / 2}\right.\right. \\
& +4 a^{3 / 2} \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{x} b-2 \ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) \sqrt{x} a b^{2}-\ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) x a^{2} b \\
& \left.\left.+2 \sqrt{a} b^{2} \sqrt{\sqrt{x}(b+a \sqrt{x})}-\ln \left(\frac{2 \sqrt{\sqrt{x}(b+a \sqrt{x})} \sqrt{a}+2 a \sqrt{x}+b}{2 \sqrt{a}}\right) b^{3}\right)\right)
\end{aligned}
$$

Problem 74: Unable to integrate problem.

$$
\int \frac{\left(b x^{3}+a x^{2}\right)^{n}}{x^{3 n}} \mathrm{~d} x
$$

Optimal(type 5, 50 leaves, 3 steps):

$$
\frac{x^{-1-3 n}\left(b x^{3}+a x^{2}\right)^{1+n} \text { hypergeom }\left([1,2],[2-n],-\frac{b x}{a}\right)}{a(1-n)}
$$

Result(type 8, 23 leaves):

$$
\int \frac{\left(b x^{3}+a x^{2}\right)^{n}}{x^{3 n}} \mathrm{~d} x
$$

Problem 79: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{13 / 2}}{\sqrt{b x^{5}+a x^{2}}} d x
$$

Optimal(type 4, 272 leaves, 5 steps):

$$
\begin{aligned}
& \frac{x^{5 / 2} \sqrt{b x^{5}+a x^{2}}}{5 b}-\frac{7 a \sqrt{b x^{5}+a x^{2}}}{20 b^{2} \sqrt{x}}+\left(7 a ^ { 5 / 3 } x ^ { 3 / 2 } ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.\quad+\sqrt{3})) \text { EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) /\left(1 2 0 \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.-\sqrt{3})) b^{2} \sqrt{b x^{5}+a x^{2}} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

Result(type ?, 2016 leaves): Display of huge result suppressed!
Problem 80: Result more than twice size of optimal antiderivative.

$$
\int \frac{x^{5} / 2}{\sqrt{b x^{5}+a x^{2}}} d x
$$

Optimal(type 4, 512 leaves, 5 steps):

$$
\begin{aligned}
& \frac{x^{3 / 2}\left(b x^{3}+a\right)(1+\sqrt{3})}{b^{2 / 3}\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right) \sqrt{b x^{5}+a x^{2}}}-\left(3 ^ { 1 / 4 } a ^ { 1 / 3 } x ^ { 3 / 2 } ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}+b^{1 / 3} x(1\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\sqrt{3})) b^{2 / 3} \sqrt{b x^{5}+a x^{2}} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)-\left(a ^ { 1 / 3 } x ^ { 3 / 2 } ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1+\sqrt{3})\right) \text { EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)(1-\sqrt{3}) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) / \\
& \left(6\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right) b^{2 / 3} \sqrt{b x^{5}+a x^{2}} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

Result(type ?, 2373 leaves): Display of huge result suppressed!
Problem 81: Result more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{x} \sqrt{b x^{5}+a x^{2}}} \mathrm{~d} x
$$

Optimal(type 4, 537 leaves, 6 steps):

$$
\begin{aligned}
& \frac{2 b^{1 / 3} x^{3 / 2}\left(b x^{3}+a\right)(1+\sqrt{3})}{a\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right) \sqrt{b x^{5}+a x^{2}}}-\frac{2 \sqrt{b x^{5}+a x^{2}}}{a x^{3 / 2}}-\left(2 3 ^ { 1 / 4 } b ^ { 1 / 3 } x ^ { 3 / 2 } ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1+\sqrt{3})\right) \text { EllipticE }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right) /\left(\left(a^{1 / 3}+b^{1 / 3} x(1\right.\right. \\
& \left.-\sqrt{3})) a^{2 / 3} \sqrt{b x^{5}+a x^{2}} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)-\left(b ^ { 1 / 3 } x ^ { 3 / 2 } ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ) \sqrt { \frac { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 - \sqrt { 3 } ) ) ^ { 2 } } { ( a ^ { 1 / 3 } + b ^ { 1 / 3 } x ( 1 + \sqrt { 3 } ) ) ^ { 2 } } } \left(a^{1 / 3}\right.\right. \\
& \left.\left.+b^{1 / 3} x(1+\sqrt{3})\right) \text { EllipticF }\left(\sqrt{1-\frac{\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right)^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}, \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)(1-\sqrt{3}) \sqrt{\frac{a^{2 / 3}-a^{1 / 3} b^{1 / 3} x+b^{2 / 3} x^{2}}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}} 3^{3 / 4}\right) / \\
& \left(3\left(a^{1 / 3}+b^{1 / 3} x(1-\sqrt{3})\right) a^{2 / 3} \sqrt{b x^{5}+a x^{2}} \sqrt{\frac{b^{1 / 3} x\left(a^{1 / 3}+b^{1 / 3} x\right)}{\left(a^{1 / 3}+b^{1 / 3} x(1+\sqrt{3})\right)^{2}}}\right)
\end{aligned}
$$

Result(type ?, 2859 leaves): Display of huge result suppressed!
Problem 86: Result more than twice size of optimal antiderivative.

$$
\int x^{24}\left(b x^{38}+a x\right)^{12} \mathrm{~d} x
$$

Optimal(type 1, 14 leaves, 2 steps):

$$
\frac{\left(b x^{37}+a\right)^{13}}{481 b}
$$

Result(type 1, 134 leaves):

$$
\begin{aligned}
& \frac{1}{481} b^{12} x^{481}+\frac{1}{37} a b^{11} x^{444}+\frac{6}{37} a^{2} b^{10} x^{407}+\frac{22}{37} a^{3} b^{9} x^{370}+\frac{55}{37} a^{4} b^{8} x^{333}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{132}{37} a^{7} b^{5} x^{222}+\frac{99}{37} a^{8} b^{4} x^{185} \\
& \quad+\frac{55}{37} a^{9} b^{3} x^{148}+\frac{22}{37} a^{10} b^{2} x^{111}+\frac{6}{37} a^{11} b x^{74}+\frac{1}{37} a^{12} x^{37}
\end{aligned}
$$

Problem 87: Result more than twice size of optimal antiderivative.

$$
\int\left(b x^{14}+a x\right)^{12} \mathrm{~d} x
$$

Optimal(type 1, 14 leaves, 2 steps):

$$
\frac{\left(b x^{13}+a\right)^{13}}{169 b}
$$

Result(type 1, 134 leaves):


$$
+\frac{22}{13} a^{10} b^{2} x^{39}+\frac{6}{13} a^{11} b x^{26}+\frac{1}{13} a^{12} x^{13}
$$

Problem 88: Result more than twice size of optimal antiderivative.

$$
\int\left(b x^{27}+a x^{2}\right)^{12} \mathrm{~d} x
$$

Optimal(type 1, 14 leaves, 2 steps):

$$
\frac{\left(b x^{25}+a\right)^{13}}{325 b}
$$

Result(type 1, 134 leaves):
$\frac{1}{325} b^{12} x^{325}+\frac{1}{25} a b^{11} x^{300}+\frac{6}{25} a^{2} b^{10} x^{275}+\frac{22}{25} a^{3} b^{9} x^{250}+\frac{11}{5} a^{4} b^{8} x^{225}+\frac{99}{25} a^{5} b^{7} x^{200}+\frac{132}{25} a^{6} b^{6} x^{175}+\frac{132}{25} a^{7} b^{5} x^{150}+\frac{99}{25} a^{8} b^{4} x^{125}$ $+\frac{11}{5} a^{9} b^{3} x^{100}+\frac{22}{25} a^{10} b^{2} x^{75}+\frac{6}{25} a^{11} b x^{50}+\frac{1}{25} a^{12} x^{25}$

Problem 89: Result more than twice size of optimal antiderivative.

$$
\int\left(a x^{m}+b x^{1+13 m}\right)^{12} \mathrm{~d} x
$$

Optimal(type 3, 25 leaves, 2 steps):

$$
\frac{\left(a+b x^{1+12 m}\right)^{13}}{13 b(1+12 m)}
$$

Result(type 3, 286 leaves):

$$
\begin{aligned}
& \frac{b^{12} x^{13}\left(x^{m}\right)^{156}}{13(1+12 m)}+\frac{a b^{11} x^{12}\left(x^{m}\right)^{144}}{1+12 m}+\frac{6 a^{2} b^{10} x^{11}\left(x^{m}\right)^{132}}{1+12 m}+\frac{22 a^{3} b^{9} x^{10}\left(x^{m}\right)^{120}}{1+12 m}+\frac{55 a^{4} b^{8} x^{9}\left(x^{m}\right)^{108}}{1+12 m}+\frac{99 a^{5} b^{7} x^{8}\left(x^{m}\right)^{96}}{1+12 m}+\frac{132 a^{6} b^{6} x^{7}\left(x^{m}\right)^{84}}{1+12 m} \\
& \quad+\frac{132 a^{7} b^{5} x^{6}\left(x^{m}\right)^{72}}{1+12 m}+\frac{99 a^{8} b^{4} x^{5}\left(x^{m}\right)^{60}}{1+12 m}+\frac{55 a^{9} b^{3} x^{4}\left(x^{m}\right)^{48}}{1+12 m}+\frac{22 a^{10} b^{2} x^{3}\left(x^{m}\right)^{36}}{1+12 m}+\frac{6 a^{11} b x^{2}\left(x^{m}\right)^{24}}{1+12 m}+\frac{a^{12} x\left(x^{m}\right)^{12}}{1+12 m}
\end{aligned}
$$

Problem 95: Result more than twice size of optimal antiderivative.

$$
\int x^{p}\left(a x^{n}+b x^{1+13 n+p}\right)^{12} \mathrm{~d} x
$$

Optimal(type 3, 27 leaves, 2 steps):

$$
\frac{\left(a+b x^{1+12 n+p}\right)^{13}}{13 b(1+12 n+p)}
$$

Result(type 3, 362 leaves):

$$
\begin{aligned}
& \frac{b^{12} x^{13}\left(x^{n}\right)^{156}\left(x^{p}\right)^{13}}{13(1+12 n+p)}+\frac{a b^{11} x^{12}\left(x^{n}\right)^{144}\left(x^{p}\right)^{12}}{1+12 n+p}+\frac{6 a^{2} b^{10} x^{11}\left(x^{n}\right)^{132}\left(x^{p}\right)^{11}}{1+12 n+p}+\frac{22 a^{3} b^{9} x^{10}\left(x^{n}\right)^{120}\left(x^{p}\right)^{10}}{1+12 n+p}+\frac{55 a^{4} b^{8} x^{9}\left(x^{n}\right)^{108}\left(x^{p}\right)^{9}}{1+12 n+p} \\
& \quad+\frac{99 a^{5} b^{7} x^{8}\left(x^{n}\right)^{96}\left(x^{p}\right)^{8}}{1+12 n+p}+\frac{132 a^{6} b^{6} x^{7}\left(x^{n}\right)^{84}\left(x^{p}\right)^{7}}{1+12 n+p}+\frac{132 a^{7} b^{5} x^{6}\left(x^{n}\right)^{72}\left(x^{p}\right)^{6}}{1+12 n+p}+\frac{99 a^{8} b^{4} x^{5}\left(x^{n}\right)^{60}\left(x^{p}\right)^{5}}{1+12 n+p}+\frac{55 a^{9} b^{3} x^{4}\left(x^{n}\right)^{48}\left(x^{p}\right)^{4}}{1+12 n+p} \\
& \quad+\frac{22 a^{10} b^{2} x^{3}\left(x^{n}\right)^{36}\left(x^{p}\right)^{3}}{1+12 n+p}+\frac{6 a^{11} b x^{2}\left(x^{n}\right)^{24}\left(x^{p}\right)^{2}}{1+12 n+p}+\frac{a^{12} x\left(x^{n}\right)^{12} x^{p}}{1+12 n+p}
\end{aligned}
$$

Problem 96: Result more than twice size of optimal antiderivative.

$$
\int x^{12}\left(b x^{13}+a\right)^{12} \mathrm{~d} x
$$

Optimal(type 1, 14 leaves, 1 step):

$$
\frac{\left(b x^{13}+a\right)^{13}}{169 b}
$$

Result(type 1, 134 leaves):
$\frac{1}{169} b^{12} x^{169}+\frac{1}{13} a b^{11} x^{156}+\frac{6}{13} a^{2} b^{10} x^{143}+\frac{22}{13} a^{3} b^{9} x^{130}+\frac{55}{13} a^{4} b^{8} x^{117}+\frac{99}{13} a^{5} b^{7} x^{104}+\frac{132}{13} a^{6} b^{6} x^{91}+\frac{132}{13} a^{7} b^{5} x^{78}+\frac{99}{13} a^{8} b^{4} x^{65}+\frac{55}{13} a^{9} b^{3} x^{52}$

$$
+\frac{22}{13} a^{10} b^{2} x^{39}+\frac{6}{13} a^{11} b x^{26}+\frac{1}{13} a^{12} x^{13}
$$

Problem 97: Result more than twice size of optimal antiderivative.

$$
\int x^{36}\left(b x^{37}+a\right)^{12} \mathrm{~d} x
$$

Optimal(type 1, 14 leaves, 1 step):

$$
\frac{\left(b x^{37}+a\right)^{13}}{481 b}
$$

Result(type 1, 134 leaves):
$\frac{1}{481} b^{12} x^{481}+\frac{1}{37} a b^{11} x^{444}+\frac{6}{37} a^{2} b^{10} x^{407}+\frac{22}{37} a^{3} b^{9} x^{370}+\frac{55}{37} a^{4} b^{8} x^{333}+\frac{99}{37} a^{5} b^{7} x^{296}+\frac{132}{37} a^{6} b^{6} x^{259}+\frac{132}{37} a^{7} b^{5} x^{222}+\frac{99}{37} a^{8} b^{4} x^{185}$
$+\frac{55}{37} a^{9} b^{3} x^{148}+\frac{22}{37} a^{10} b^{2} x^{111}+\frac{6}{37} a^{11} b x^{74}+\frac{1}{37} a^{12} x^{37}$

Problem 99: Unable to integrate problem.

$$
\int x^{-1-\frac{j}{2}} \sqrt{a x^{j}+b x^{n}} \mathrm{~d} x
$$

Optimal(type 3, 65 leaves, 3 steps):

$$
\frac{2 \operatorname{arctanh}\left(\frac{x^{\frac{j}{2}} \sqrt{a}}{\sqrt{a x^{j}+b x^{n}}}\right) \sqrt{a}}{j-n}-\frac{2 \sqrt{a x^{j}+b x^{n}}}{(j-n) x^{\frac{j}{2}}}
$$

Result(type 8, 23 leaves):

$$
\int x^{-1-\frac{j}{2}} \sqrt{a x^{j}+b x^{n}} \mathrm{~d} x
$$

Problem 101: Unable to integrate problem.

$$
\int \frac{\left(a x^{2}+b x^{n}\right)^{3 / 2}}{c^{4} x^{4}} d x
$$

Optimal(type 3, 92 leaves, 5 steps):

$$
-\frac{2\left(a x^{2}+b x^{n}\right)^{3 / 2}}{3 c^{4}(2-n) x^{3}}+\frac{2 a^{3 / 2} \operatorname{arctanh}\left(\frac{x \sqrt{a}}{\sqrt{a x^{2}+b x^{n}}}\right)}{c^{4}(2-n)}-\frac{2 a \sqrt{a x^{2}+b x^{n}}}{c^{4}(2-n) x}
$$

Result(type 8, 68 leaves):

$$
\frac{2\left(4 a x^{2}+b \mathrm{e}^{n \ln (x)}\right) \sqrt{a x^{2}+b \mathrm{e}^{n \ln (x)}}}{3(n-2) x^{3} c^{4}}+\frac{\int \frac{a^{2}}{\sqrt{a x^{2}+b \mathrm{e}^{n \ln (x)}}} \mathrm{d} x}{c^{4}}
$$

Problem 102: Unable to integrate problem.

$$
\int(c x)^{7 / 2}\left(\frac{a}{x^{3}}+b x^{n}\right)^{3 / 2} \mathrm{~d} x
$$

Optimal(type 3, 100 leaves, 5 steps):

$$
\frac{2(c x)^{9 / 2}\left(\frac{a}{x^{3}}+b x^{n}\right)^{3 / 2}}{3 c(3+n)}-\frac{2 a^{3 / 2} c^{4} \operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3 / 2} \sqrt{\frac{a}{x^{3}}+b x^{n}}}\right) \sqrt{x}}{(3+n) \sqrt{c x}}+\frac{2 a c^{2}(c x)^{3 / 2} \sqrt{\frac{a}{x^{3}}+b x^{n}}}{3+n}
$$

Result(type 8, 21 leaves):

$$
\int(c x)^{7 / 2}\left(\frac{a}{x^{3}}+b x^{n}\right)^{3 / 2} \mathrm{~d} x
$$

Problem 106: Unable to integrate problem.

$$
\int \frac{(c x)^{-1+\frac{j}{2}}}{\sqrt{a x^{j}+b x^{n}}} \mathrm{~d} x
$$

Optimal(type 3, 52 leaves, 3 steps):

$$
\frac{2(c x)^{\frac{j}{2}} \operatorname{arctanh}\left(\frac{x^{\frac{j}{2}} \sqrt{a}}{\sqrt{a x^{j}+b x^{n}}}\right)}{c(j-n) x^{\frac{j}{2}} \sqrt{a}}
$$

$$
\int \frac{(c x)^{-1+\frac{j}{2}}}{\sqrt{a x^{j}+b x^{n}}} \mathrm{~d} x
$$

Problem 107: Unable to integrate problem.

$$
\int \frac{1}{\sqrt{a x^{2}+b x^{n}}} \mathrm{~d} x
$$

Optimal(type 3, 31 leaves, 2 steps):

$$
\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{a}}{\sqrt{a x^{2}+b x^{n}}}\right)}{(2-n) \sqrt{a}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\sqrt{a x^{2}+b x^{n}}} d x
$$

Problem 108: Unable to integrate problem.

$$
\int \frac{1}{(c x)^{5 / 2}\left(\frac{a}{x}+b x^{n}\right)^{3 / 2}} \mathrm{~d} x
$$

Optimal(type 3, 74 leaves, 4 steps):

$$
-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x} \sqrt{\frac{a}{x}+b x^{n}}}\right) \sqrt{x}}{a^{3 / 2} c^{2}(1+n) \sqrt{c x}}+\frac{2}{a c^{2}(1+n) \sqrt{c x} \sqrt{\frac{a}{x}+b x^{n}}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{1}{(c x)^{5 / 2}\left(\frac{a}{x}+b x^{n}\right)^{3 / 2}} \mathrm{~d} x
$$

Problem 109: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$
\int \frac{1}{\sqrt{\frac{b x^{3}+a}{x}}} \mathrm{~d} x
$$

Optimal(type 3, 24 leaves, 3 steps):

$$
\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{\frac{a}{x}+b x^{2}}}\right)}{3 \sqrt{b}}
$$

Result(type 4, 476 leaves):
$\left(4\left(b x^{3}+a\right)(\mathrm{I} \sqrt{3}-1) \sqrt{-\frac{(\mathrm{I} \sqrt{3}-3) x b}{(\mathrm{I} \sqrt{3}-1)\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)}}\left(-b x+\left(-a b^{2}\right)^{1 /}\right.\right.$
$3^{3} \sqrt{\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}+2 b x+\left(-a b^{2}\right)^{1 / 3}}{(\mathrm{I} \sqrt{3}+1)\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)}} \sqrt{\frac{\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}-2 b x-\left(-a b^{2}\right)^{1 / 3}}{(\mathrm{I} \sqrt{3}-1)\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)}}\left(\right.$ EllipticPi $\sqrt{-\frac{(\mathrm{I} \sqrt{3}-3) x b}{(\mathrm{I} \sqrt{3}-1)\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)}}$,
$\left.\frac{\mathrm{I} \sqrt{3}-1}{\mathrm{I} \sqrt{3}-3}, \sqrt{\frac{(\mathrm{I} \sqrt{3}+3)(\mathrm{I} \sqrt{3}-1)}{(\mathrm{I} \sqrt{3}+1)(\mathrm{I} \sqrt{3}-3)}}\right)-$ EllipticF $\left.\left.\left(\sqrt{-\frac{(\mathrm{I} \sqrt{3}-3) x b}{(\mathrm{I} \sqrt{3}-1)\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)}}, \sqrt{\frac{(\mathrm{I} \sqrt{3}+3)(\mathrm{I} \sqrt{3}-1)}{(\mathrm{I} \sqrt{3}+1)(\mathrm{I} \sqrt{3}-3)}}\right)\right) /\right) /$
$\left(b^{2} \sqrt{\frac{b x^{3}+a}{x}} \sqrt{x\left(b x^{3}+a\right)}(\mathrm{I} \sqrt{3}\right.$
$\left.-3) \sqrt{\frac{x\left(-b x+\left(-a b^{2}\right)^{1 / 3}\right)\left(\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}+2 b x+\left(-a b^{2}\right)^{1 / 3}\right)\left(\mathrm{I} \sqrt{3}\left(-a b^{2}\right)^{1 / 3}-2 b x-\left(-a b^{2}\right)^{1 / 3}\right)}{b^{2}}}\right)$

Problem 110: Unable to integrate problem.

$$
\int \frac{1}{\sqrt{x^{n}\left(a+b x^{2-n}\right)}} \mathrm{d} x
$$

Optimal(type 3, 31 leaves, 3 steps):

$$
\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{b x^{2}+a x^{n}}}\right)}{(2-n) \sqrt{b}}
$$

Result(type 8, 19 leaves):

$$
\int \frac{1}{\sqrt{x^{n}\left(a+b x^{2-n}\right)}} \mathrm{d} x
$$

Problem 111: Unable to integrate problem.

$$
\int \frac{1}{\sqrt{x\left(b x+a x^{-1+n}\right)}} \mathrm{d} x
$$

Optimal (type 3, 31 leaves, 3 steps):

$$
\frac{2 \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{b x^{2}+a x^{n}}}\right)}{(2-n) \sqrt{b}}
$$

$$
\int \frac{1}{\sqrt{x\left(b x+a x^{-1+n}\right)}} \mathrm{d} x
$$

Problem 112: Unable to integrate problem.

$$
\int(c x)^{m} \sqrt{a x^{j}+b x^{n}} \mathrm{~d} x
$$

Optimal(type 5, 94 leaves, 3 steps):

$$
\frac{2 x(c x)^{m} \text { hypergeom }\left(\left[-\frac{1}{2}, \frac{1+m+\frac{n}{2}}{j-n}\right],\left[1+\frac{2+2 m+n}{2 j-2 n}\right],-\frac{a x^{j-n}}{b}\right) \sqrt{a x^{j}+b x^{n}}}{(2+2 m+n) \sqrt{1+\frac{a x^{j-n}}{b}}}
$$

Result(type 8, 161 leaves):
$\frac{2 x \mathrm{e}^{m\left(\ln (c)+\ln (x)-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} c x)(-\operatorname{csgn}(\mathrm{I} c x)+\operatorname{csgn}(\mathrm{I} c))(-\operatorname{csgn}(\mathrm{I} c x)+\operatorname{csgn}(\mathrm{I} x))}{2}\right) \sqrt{a \mathrm{e}^{j \ln (x)}+b \mathrm{e}^{n \ln (x)}}}+\int 2+2 m+n}{2+1}$

$$
-\frac{\mathrm{e}^{m\left(\ln (c)+\ln (x)-\frac{\mathrm{I} \pi \operatorname{csgn}(\mathrm{I} c x)(-\operatorname{csgn}(\mathrm{I} c x)+\operatorname{csgn}(\mathrm{I} c))(-\operatorname{csgn}(\mathrm{I} c x)+\operatorname{csgn}(\mathrm{I} x))}{2}\right)} a \mathrm{e}^{j \ln (x)}(j-n)}{(2+2 m+n) \sqrt{a \mathrm{e}^{j \ln (x)}+b \mathrm{e}^{n \ln (x)}}} \mathrm{d} x
$$

Problem 113: Unable to integrate problem.

$$
\int \frac{(c x)^{m}}{\left(a x^{j}+b x^{n}\right)^{5 / 2}} \mathrm{~d} x
$$

Optimal(type 5, 103 leaves, 3 steps):

$$
\frac{2 x^{1-2 n}(c x)^{m} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{1+m-\frac{5 n}{2}}{j-n}\right],\left[1+\frac{1+m-\frac{5 n}{2}}{j-n}\right],-\frac{a x^{j-n}}{b}\right) \sqrt{1+\frac{a x^{j-n}}{b}}}{b^{2}(2+2 m-5 n) \sqrt{a x^{j}+b x^{n}}}
$$

Result(type 8, 21 leaves):

$$
\int \frac{(c x)^{m}}{\left(a x^{j}+b x^{n}\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 114: Unable to integrate problem.

$$
\int \frac{1}{\left(a x^{j}+b x^{n}\right)^{5 / 2}} d x
$$

Optimal(type 5, 95 leaves, 3 steps):

$$
\frac{2 x^{1-2 n} \text { hypergeom }\left(\left[\frac{5}{2}, \frac{1-\frac{5 n}{2}}{j-n}\right],\left[1+\frac{2-5 n}{2 j-2 n}\right],-\frac{a x^{j-n}}{b}\right) \sqrt{1+\frac{a x^{j-n}}{b}}}{b^{2}(2-5 n) \sqrt{a x^{j}+b x^{n}}}
$$

Result(type 8, 15 leaves):

$$
\int \frac{1}{\left(a x^{j}+b x^{n}\right)^{5 / 2}} \mathrm{~d} x
$$

Problem 118: Unable to integrate problem.

$$
\int \frac{1}{\left(a x^{1 / 3}+b x^{2 / 3}\right)^{1 / 3}} \mathrm{~d} x
$$

Optimal (type 4, 738 leaves, 11 steps):

$$
\begin{aligned}
& -\frac{45 a\left(a+b x^{1 / 3}\right) x^{1 / 3}}{28 b^{2}\left(a x^{1 / 3}+b x^{2 / 3}\right)^{1 / 3}}+\frac{9\left(a+b x^{1 / 3}\right) x^{2 / 3}}{7 b\left(a x^{1 / 3}+b x^{2 / 3}\right)^{1 / 3}}-\frac{45 a^{2}\left(a+2 b x^{1 / 3}\right)\left(-\frac{b\left(a x^{1 / 3}+b x^{2 / 3}\right)}{a^{2}}\right)^{1 / 3} 2^{2 / 3}}{28 b^{3}\left(a x^{1 / 3}+b x^{2 / 3}\right)^{1 / 3}\left(1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}-\sqrt{3}\right)} \\
& +\left(153^{3 / 4} a^{4}\left(1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}\right)\left(-\frac{b\left(a x^{1 / 3}+b x^{2 / 3}\right)}{a^{2}}\right)^{1 / 3} \operatorname{EllipticF}\left(\frac{1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}+\sqrt{3}}{1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}-\sqrt{3}}, 2 \mathrm{I}-\mathrm{I} \sqrt{3}\right)\right. \\
& \left.+b x^{2 / 3}\right)^{1 / 3} \sqrt{\left.\frac{-1+2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}}{\left(1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}-\sqrt{3}\right)^{2}}\right)-\left(453^{1 / 4} a^{4}\left(1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}\right)\left(-\frac{b\left(a x^{1 / 3}+b x^{2 / 3}\right)}{a^{2}}\right)\right.} \\
& \left.\sqrt[3]{\frac{-1+2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}}{\left(1-2^{2 / 3}\left(-\frac{b\left(a+b x^{1 / 3}\right) x^{1 / 3}}{a^{2}}\right)^{1 / 3}-\sqrt{3}\right)^{2}}}\right) \\
& \text { Result(type 8, } 15 \text { leaves): } \\
& \int \frac{1}{\left(a x^{1 / 3}+b x^{2 / 3}\right)^{1 / 3}} \mathrm{~d} x
\end{aligned}
$$

Problem 119: Unable to integrate problem.

$$
\int x^{m}\left(a x^{j}+b x^{n}\right)^{p} \mathrm{~d} x
$$

Optimal(type 5, 91 leaves, 3 steps):

$$
\frac{x^{1+m}\left(a x^{j}+b x^{n}\right)^{p}\left(a+b x^{-j+n}\right) \text { hypergeom }\left(\left[1,1+p+\frac{j p+m+1}{-j+n}\right],\left[1+\frac{j p+m+1}{-j+n}\right],-\frac{b x^{-j+n}}{a}\right)}{a(j p+m+1)}
$$

Result(type 8, 19 leaves):

$$
\int x^{m}\left(a x^{j}+b x^{n}\right)^{p} \mathrm{~d} x
$$

Problem 120: Unable to integrate problem.

$$
\int\left(a x^{m}+b x^{m p+m+1}\right)^{p} \mathrm{~d} x
$$

Optimal (type 3, 45 leaves, 1 step):

$$
\frac{\left(a x^{m}+b x^{m p+m+1}\right)^{1+p}}{b(1+p)(m p+1) x^{m(1+p)}}
$$

Result(type 8, 20 leaves):

$$
\int\left(a x^{m}+b x^{m p+m+1}\right)^{p} \mathrm{~d} x
$$

Problem 121: Unable to integrate problem.

$$
\int x^{n}\left(a x^{m}+b x^{m p+m+n+1}\right)^{p} \mathrm{~d} x
$$

Optimal (type 3, 47 leaves, 1 step):

$$
\frac{\left(a x^{m}+b x^{m p+m+n+1}\right)^{1+p}}{b(1+p)(m p+n+1) x^{m(1+p)}}
$$

Result (type 8, 25 leaves):

$$
\int x^{n}\left(a x^{m}+b x^{m p+m+n+1}\right)^{p} \mathrm{~d} x
$$

Problem 122: Unable to integrate problem.

$$
\int\left(x^{\frac{-1+n}{p}}\left(a+b x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Optimal(type 3, 59 leaves, 2 steps):

$$
\frac{x^{\frac{(1-n)(1+p)}{p}}\left(b x^{n+\frac{-1+n}{p}}+\frac{a}{x^{\frac{1-n}{p}}}\right)^{1+p}}{b n(1+p)}
$$

$$
\int\left(x^{\frac{-1+n}{p}}\left(a+b x^{n}\right)\right)^{p} \mathrm{~d} x
$$

Problem 123: Unable to integrate problem.

$$
\int x^{-1+n-p(1+q)}\left(a x^{n}+b x^{p}\right)^{q} \mathrm{~d} x
$$

Optimal(type 3, 40 leaves, 1 step):

$$
\frac{\left(a x^{n}+b x^{p}\right)^{1+q}}{a(n-p)(1+q) x^{p(1+q)}}
$$

Result(type 8, 27 leaves):

$$
\int x^{-1+n-p(1+q)}\left(a x^{n}+b x^{p}\right)^{q} \mathrm{~d} x
$$

Test results for the 79 problems in "1.1.4.3 (e x $)^{\wedge} m\left(a x^{\wedge} j+b x^{\wedge} k\right)^{\wedge} p\left(c+d x x^{\wedge} n\right)^{\wedge} q \cdot t x t^{\prime \prime}$
Problem 38: Result more than twice size of optimal antiderivative.

$$
\int \frac{\left(B x^{2}+A\right)\left(c x^{4}+b x^{2}\right)^{3 / 2}}{x^{10}} \mathrm{~d} x
$$

Optimal(type 3, 120 leaves, 5 steps):

$$
-\frac{(-A c+6 b B)\left(c x^{4}+b x^{2}\right)^{3 / 2}}{24 b x^{7}}-\frac{A\left(c x^{4}+b x^{2}\right)^{5 / 2}}{6 b x^{11}}-\frac{c^{2}(-A c+6 b B) \operatorname{arctanh}\left(\frac{x \sqrt{b}}{\sqrt{c x^{4}+b x^{2}}}\right)}{16 b^{3 / 2}}-\frac{c(-A c+6 b B) \sqrt{c x^{4}+b x^{2}}}{16 b x^{3}}
$$

Result(type 3, 272 leaves):

$$
\begin{aligned}
& -\frac{1}{48 x^{9}\left(x^{2} c+b\right)^{3 / 2} b^{9 / 2}}\left(( c x ^ { 4 } + b x ^ { 2 } ) ^ { 3 / 2 } \left(-18 B c^{2} \sqrt{x^{2} c+b} b^{7 / 2} x^{6}-6 B c^{2}\left(x^{2} c+b\right)^{3 / 2} b^{5 / 2} x^{6}+3 A c^{3} \sqrt{x^{2} c+b} b^{5 / 2} x^{6}+A c^{3}\left(x^{2} c+b\right)^{3 /}\right.\right. \\
& { }^{2} x^{6} b^{3 / 2}+6 B c\left(x^{2} c+b\right)^{5 / 2} b^{5 / 2} x^{4}-A c^{2}\left(x^{2} c+b\right)^{5 / 2} x^{4} b^{3 / 2}+12 B\left(x^{2} c+b\right)^{5 / 2} b^{7 / 2} x^{2}-2 A c\left(x^{2} c+b\right)^{5 / 2} b^{5 / 2} x^{2} \\
& \left.\left.-3 A c^{3} \ln \left(\frac{2\left(\sqrt{b} \sqrt{x^{2} c+b}+b\right)}{x}\right) b^{3} x^{6}+18 B c^{2} \ln \left(\frac{2\left(\sqrt{b} \sqrt{x^{2} c+b}+b\right)}{x}\right) b^{4} x^{6}+8 A\left(x^{2} c+b\right)^{5 / 2} b^{7 / 2}\right)\right)
\end{aligned}
$$

Problem 76: Unable to integrate problem.

$$
\int \frac{a x^{m}+b x^{n}}{c x^{m}+d x^{n}} \mathrm{~d} x
$$

Optimal(type 5, 56 leaves, 4 steps):

$$
\frac{a x}{c}+\frac{(-d a+b c) x \text { hypergeom }\left(\left[1, \frac{1}{m-n}\right],\left[1+\frac{1}{m-n}\right],-\frac{c x^{m-n}}{d}\right)}{c d}
$$

Result(type 8, 43 leaves):

$$
\frac{b x}{d}+\int \frac{\mathrm{e}^{m \ln (x)}(d a-b c)}{\left(c \mathrm{e}^{m \ln (x)}+d \mathrm{e}^{n \ln (x)}\right) d} \mathrm{~d} x
$$

Problem 77: Unable to integrate problem.

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{d x+c} \mathrm{~d} x
$$

Optimal(type 6, 64 leaves, 4 steps):

$$
\frac{\left(a+\frac{b}{x}\right)^{n} x^{m} \text { AppellF1 }\left(-m,-n, 1,1-m,-\frac{b}{a x},-\frac{c}{d x}\right)}{d m\left(1+\frac{b}{a x}\right)^{n}}
$$

Result(type 8, 22 leaves):

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n} x^{m}}{d x+c} \mathrm{~d} x
$$

Problem 78: Unable to integrate problem.

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{2}(d x+c)} \mathrm{d} x
$$

Optimal(type 5, 86 leaves, 4 steps):

$$
-\frac{\left(a+\frac{b}{x}\right)^{1+n}}{b c(1+n)}-\frac{d\left(a+\frac{b}{x}\right)^{1+n} \text { hypergeom }\left([1,1+n],[2+n], \frac{c\left(a+\frac{b}{x}\right)}{a c-b d}\right)}{c(a c-b d)(1+n)}
$$

Result(type 8, 22 leaves):

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n}}{x^{2}(d x+c)} \mathrm{d} x
$$

Problem 79: Unable to integrate problem.

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(d x+c)^{2}} \mathrm{~d} x
$$

Optimal(type 5, 58 leaves, 3 steps):

$$
-\frac{b\left(a+\frac{b}{x}\right)^{1+n} \text { hypergeom }\left([2,1+n],[2+n], \frac{c\left(a+\frac{b}{x}\right)}{a c-b d}\right)}{(a c-b d)^{2}(1+n)}
$$

Result(type 8, 19 leaves):

$$
\int \frac{\left(a+\frac{b}{x}\right)^{n}}{(d x+c)^{2}} \mathrm{~d} x
$$

## Summary of Integration Test Results

202 integration problems


A - 158 optimal antiderivatives
B - 18 more than twice size of optimal antiderivatives
C - 4 unnecessarily complex antiderivatives
D - 22 unable to integrate problems
E - 0 integration timeouts

